MultiFat

Theory Manual

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1 Summary

The Section 2 describes the multiaxial rainflow applied in MultiFat. In the frame of fatigue analyses, this approach is used to extract the stress cycles in case of general non-proportional, variable-amplitude, multiaxial stress history. This allows computing the fatigue damage and thus assessing safe-life for structural components submitted to fully generic load spectra.

In the Section 3 a procedure is provided for analyses in the domain of low cycle fatigue where the elasto-plastic behavior of the material needs to be taken into account for the cycles extraction. The adopted methodology is based on the Neuber correction applied to a multiaxial context.

The methodology here illustrated is implemented in the software MultiFat for multiaxial fatigue calculation.

2 The multiaxial rainflow

2.1 General principle

In [Ref 1], Wang and Brown proposed a criterion to evaluate the fatigue damage under multiaxial loading. In the present document, the original method is simplified in order to use it as a tool to rainflow-compute a generic multiaxial stress-history and to obtain the cycles afterwards used to perform a fatigue damage evaluation. With respect to the original application, this allows using the multiaxial rainflow in a fully flexible context, with every kind of S-N relationship and with evry type of mean-stress correction. This can be done for both the cases of stress history remaining in the elastic field, so that there is no difference by treating stresses or strains (Section 2), and also when the stress can go in the elasto-plastic field (Section 3).

It is noteworthy to mention that when the stress history is purely uniaxial the multiaxial rainflow is equivalent to the classical rainflow of Matsuishi and Endo, [Ref 4].

In the multiaxial rainflow counting, the concept of "relative stresses" is applied, that allows extracting cycles by moving throw the stress history. The sequence is firstly reordered by putting as first point the one having maximum Von Mises stress σ_{Mises} in the whole stress-history and s cycle-counting starts at each point of the sequence. The Von Mises stress $\sigma_{Mises,rel}$ relative to the initial point of the counting will be considered as stress parameter, i.e. the Von Mises of the tensor difference between current point and initial point of the counting.

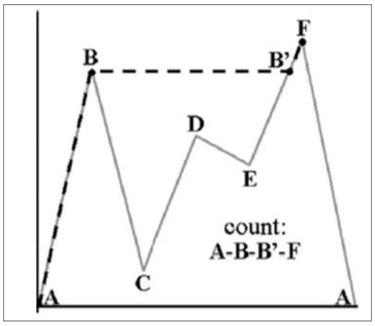

Figure 1: Path defining a cycle from [Ref 1]

The counting ends when the whole sequence has been counted. At the end of the process, all the paths of the stress history are counted one and only one time and all the cycles nested in higher amplitude cycles are recognized and extracted.

The cycles extracted have to be considered as *half-cycles* (or reversals, or inversions), as illustrated in Figure 2.

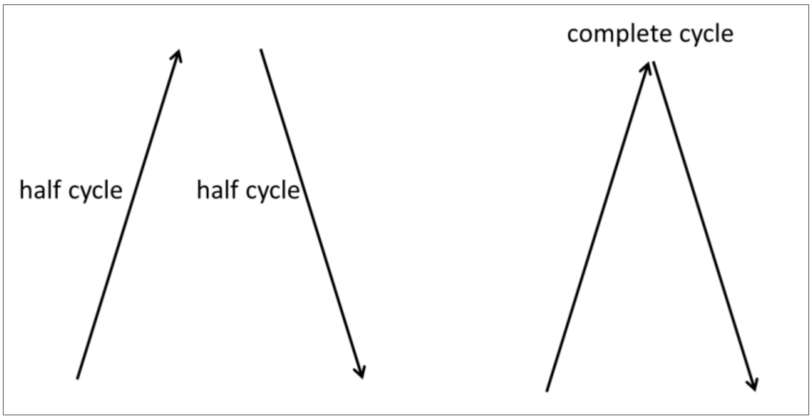

Figure 2: Half cycles (reversals)

The rainflow extracts cycles in terms of their amplitude, based on the Von Mises stress. Nevertheless, the principle could be extended to other equivalent stress criteria, like namely Tresca or Maximum Principal.

2.2 Correction of the cycles based on the stress-path

The multiaxial rainflow illustrated here above works well in the most of cases for operational applications. Nevertheless, due to the one-dimensional nature of the relative Von Mises stress used to extract the stress-cycles, these can need to be corrected as illustrated for example in the [Ref 2]. This is done in literature in several ways: for example in [Ref 5] is defined a rigorous procedure of multiaxial cycle extraction, based on stress-cycles created through the detailed definition of a mixed isotropic-kinematic hardening process. However, this method has been shown to be extremely time-consuming in its numerical application and thus quite unpractical for real cases, with relatively high number of points in the stress-history.

MultiFat makes available an automatic correction of the extracted cycles in line with the methods existing in the literature. It must be considered that two half-cycles extracted by the rainflow, yet identical in terms of extreme points σ_A, σ_B can have different stress-paths during their construction, as schematically illustrated in the Figure 3.

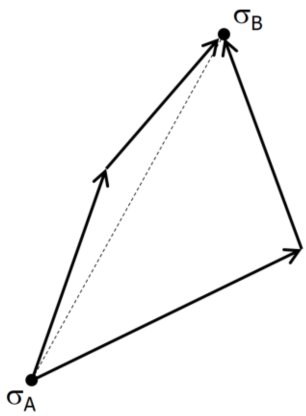

Figure 3: Illustration of the stress correction for not-straight cycles

To take into account such situations, a modification of the cycles is applied, in a way that considers somehow the path history. In the [Ref 2] and [Ref 3], different methodologies are mentioned to introduce such a correction. The one implemented in MultiFat consists for a given cycle into considering the maximum sum of the two Von Mises stresses among all the segments defined by the two extrema and by any intermediate points used for the cycle construction.

2.3 Effects of the mean stress

The influence of the mean stress has to be considered. This is done by using the classical assumption of Sines (Ref. [3]) which states to consider the mean value of the first invariant of the stress tensor, obtained during the creation of each cycle (see [Ref 5]).

$$\sigma_{mean} = \frac{1}{\sigma_a} \int_0^{\sigma_a} 3p_h(\sigma'_a)\, d\sigma'_a$$

where p_h is the hydrostatic pressure of the stress tensor $\underline{\underline{\sigma}}$:

$$p_h = \frac{1}{3} \operatorname{tr}(\underline{\underline{\sigma}})$$

and σ_a is the amplitude of a given cycle. The integral must be calculated for every cycle during its creation process. The combination of σ_a and σ_{mean} is then used for the calculation of the fatigue damage associated to a given cycle, according to a selected Mean Stress Correction like for example Goodman or Gerber.

The usage of the hydrostatic pressure as driving parameter for the influence of the mean stress is well adapted to analyses where no plasticity occurs or with slight plasticity, as evidenced by literature data. Conversely, for systematic significant plasticity cycles in the stress history, the influence of the stress quantity that is also driving the static failure (for example the Von Mises stress), increases progressively. This can be seen from test data showing namely the impact of a pure shear mean stress (thus with $p_h = 0$) on the fatigue strength, as shown in the Figure 4 below.

A methodology where this effect is taken into account is under development for the next versions of MultiFat.

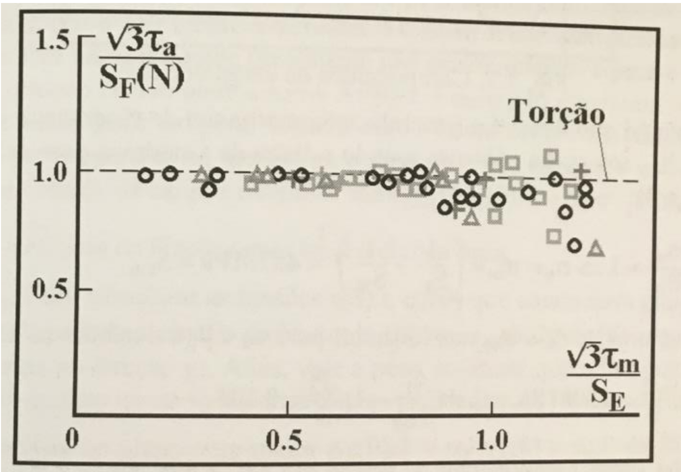

Figure 4: Effect of the mean stress for pure torsion, from [Ref 7]

3 Elastoplastic correction

3.1 General principle

In some cases, it can be needed to take into account the behavior of a material beyond its elastic limit. This section presents the approach implemented in MultiFat for considering the elastoplastic correction in a notched detail in a simple way and thus to perform cyclic elastoplastic analyses starting from linear elastic stress-analysis. This is done by means of the classical approach of Neuber. Concerning the validity of Neuber's hypotheses (confined plasticity, …) the Reader can refer to some classical book about theory of plasticity as for example the [Ref 6].

As input, a linear elastic stress e.g. calculated by a stress analysis, is provided in the form of a time history. It is noteworthy that all considerations made here are only for small-deformation plasticity theory.

The following definitions apply, with the index 'j' indicating the j^{th} element in the time history

- $\underline{\underline{\sigma}}_{EL,j}$ is the linear elastic stress
- $\underline{\underline{\sigma}}_{j}$ is the elastoplastic-corrected actual stress
- $\underline{\underline{X}}_{j}$ is the "backstress", center of the yield surface in the stress space
- σ_Y is the yield stress of the material

To apply the elasto-plastic correction, a hypothesis of Linear Kinematic Hardening (LKH) during cycling is made.

The Figure 5 illustrates the general principle of application of the elastoplastic correction rule in a multiaxial context and with LKH, between two subsequent steps j and j+1 in a stress time-history.

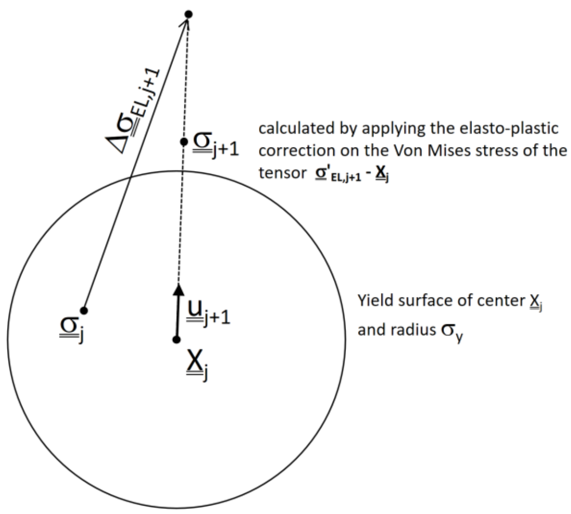

Figure 5: Elasto-plastic correction for a multiaxial cyclic stress

3.2 Neuber rule

The Neuber relationship is based on the equivalence of the two areas represented in Figure 6.

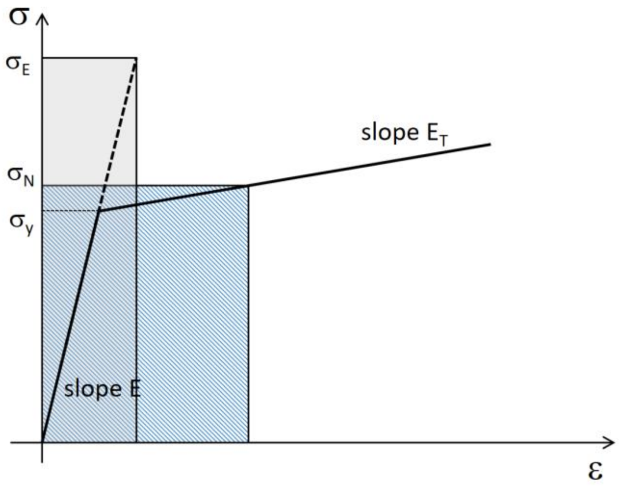

Figure 6: Graphical representation of the Neuber correction

In MultiFat, the Neuber correction can be applied in two forms:

Linearized Neuber: in this form, the classical Neuber method is linearized to its secant behaviour between the yield stress σ_Y and the static-failure stress σ_U. This approach results to have a small conservativism compared to the Neuber approach, since the deduced stresses and strains are systematically slightly higher with the linearized equation.

Classical Neuber: this form corresponds to the conventional Neuber method, see for example [Ref 7].

The User can chose arbitrarily one of the two methodologies. In MultiFat, it is recommended to use the Linearized Neuber method, due to its robustness, computational rapidity and its systematic conservativism. The two methods give anyway very close results.

4 References

[Ref 1] Wang C., Brown M., Life prediction techniques for variable amplitude multiaxial fatigue - Part 1: Theories. J Eng. Mater Technol. (1996)

[Ref 2] Marco Antonio Meggiolaro, Jaime Tupiassú Pinho de Castro, An improved multiaxial rainflow algorithm for non-proportional stress or strain histories – Part I: Enclosing surface methods - International Journal of Fatigue 42 (2012) pp. 217-226

[Ref 3] Marco Antonio Meggiolaro, Jaime Tupiassú Pinho de Castro, An improved multiaxial rainflow algorithm for non-proportional stress or strain histories – Part II: The Modified Wang-Brown method - International Journal of Fatigue 42 (2012) pp. 194-206

[Ref 4] Matsuishi M., Endo T. (1968), Fatigue of metals subjected to varying stress, Japan Soc. Mech. Engineering

[Ref 5] Malnati M., A method for calculation of finite fatigue life under multiaxial loading in high-cycle domain, Frattura ed Integrità Strutturale 28 (2014) pp. 12-18

[Ref 6] Lemaitre J., Chaboche J.-L., Mécanique des matériaux solides, Dunod, Paris, 2e édition (2001)

[Ref 7] Fadiga - Técnicas e Práticas de Dimensionamento Estrutural sob Cargas Reais de Serviço: Volume I - Iniciação de Trincas: 1